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ON THE HYDROGENIC AND NUCLEAR PROPERTIES LINK

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Abstract. *The research aims to link the nucleon's mass radius and hydrogen's ionization energy and determine the mass radii of the nucleons and some leptons. The nuclear radius of any elementary particle (R_n) is linked to atomic properties such as the ionization energy of hydrogen. It appears R_n is inversely proportional to the ionization energy of hydrogen and directly proportional to the rest - mass of the particle.*

Keywords*: Hydrogenic ions, nucleon and Bohr's radii, hydrogenic ionization energy.*

Introduction

Atomic and nuclear matter physics give that atomic properties are a direct function of the nuclear properties. For instance, the proton and neutron root mean square radii is defined as [1]

$$R_{\mathrm{n(P)}}^{\mathrm{r.m.s}} = \langle r_{\mathrm{n(P)}}^2 \rangle^{½} = \left(\frac{\int r_{\mathrm{n(P)}}^2 \rho_{n(P)}(\acute{r})\, d\acute{r}}{\int \rho_{n(P)}(\acute{r})\, d\acute{r}}\right)^{½} \approx \left(\frac{3}{5} R_{\mathrm{on(P)}}^2 + \frac{7\pi^2}{5} a_{\mathrm{n(P)}}^2\right)^{½} \quad (1)$$

Where, $R_{\mathrm{on(P)}}$, $a_{\mathrm{n(P)}}$, and $\rho_{n(P)}(\acute{r})$ are the half-density radius, diffuseness of the neutron (proton) density distributions, and local proton (neutron) density respectively.

Besides, Eq. (1) gives the impression that r.m.s. radius and by extension mass radii of proton and neutron are the same. The issue of mathematical complexity in almost all theoretical and experimental theses on elementary particle properties is very much contrary to Bohr's theory, which had met uncomplimentary remarks as discussed in the literature [2]. This has led to wave/quantum mechanics, whose mathematical formalism is too complex and sometimes lacks a stepwise approach in the derivational process [3].

In this research, a classical model is adopted.

Thus, this part of the introductory section can be described as an overview of the literature pieces of information about issues connected with nuclear matter, proton density distribution, and neutron halo or neutron skin thickness [1, 4]. Physical concepts or terms as above including surface nucleons, which are hardly explained ought to be exhaustively defined. A nonprofessional may give the impression that the nucleus is an enclosed 3-D space with internal (or intra-) nuclear nucleons and external or surface nucleons.

Atomic physics and nuclear matter physics are often exclusively treated or studied. However, atomic properties are a direct function of the nuclear properties. For instance, the nuclear charge density is a function of the total number of protons and the nuclear volume. Thus, the modern definitions of periodic law such as [5] -

(a) the properties of the elements are a periodic function of their atomic numbers.

(b) the properties of the elements depend upon their total electron configuration is the very evidence-based reason why there is a need to link nuclear matter properties with atomic matter properties.

The stability of the electronic configuration expressed in terms of ionization energy is a function of the nuclear properties. Incidentally, there seems to be emerging interest in what is referred to as the density distribution of finite nuclei as if to imply that infinite nuclei may exist [6]. Any investigation on the density distribution of the so-called finite nuclei requires according to [1] accurate information about the root-mean-square (r.m.s.) radii of proton and neutron density distribution, surface diffuseness, and neutron skin thickness. The diffuseness parameter is defined as the fall-off of the nuclear potential in the tail region of the Coulomb barrier [7]. Thus, this research is undertaken to

1) link the nuclear property and mass-radius of the nucleon to the ionization energy of hydrogen via the derivation of the appropriate equation, and

2) determine the mass radii of the nucleons and some leptons.

The size-ionization energy link

The research [8] has shown that the mass-radius of the nucleons and subatomic particles whose mass is greater than the mass of the nucleons can be determined. The greater the mass of the different particles, the longer the lengths of the mass radii become. The equation is given [8]

$$\mathcal{R}_{\mathrm{P}} = \frac{e^6\, m_{\mathrm{P}}}{4\,\pi\,\varepsilon_0^3\, m_e^2\, h^2 c^4} \tag{2}$$

Where, $\mathcal{R}_P$, e, m_P, ε_0, m_e, h, and c is the mass-radius of any particle, the charge of an electron, the mass of the particle, the permittivity in free space, the mass of an electron, Planck constant, and velocity of light in a vacuum respectively.

The Eq. (1) for c_2 gives

$$c^2 = \frac{e^3}{m_e\, h\, \varepsilon_0^{3/2}} \times \sqrt[2]{\frac{m_{\mathrm{P}}}{4\pi\mathcal{R}_{\mathrm{P}}}} \tag{3}$$

There exists the following equation for the radius of any atom [9],

$$a_{\mathrm{x}} = \frac{1}{m_e}\sqrt[2]{\frac{\xi_{\mathrm{H}}}{\xi_{\mathrm{x}}}}\;\frac{n_{\mathrm{x}}\, h^2}{\mu_0\, e^2 \pi\, c^2} \tag{4}$$

Where subscript x denotes any element of an atom, and a_x, ξ_x, n_x, μ_0, and ξ_H are the radius, the ionization energy, the principal quantum number of any atom other than the hydrogen atom, the magnetic constant, and the ionization energy of hydrogen atom respectively. If $\xi_x = \xi_H$, the usual Bohr's symbol, a_0 for the radius of hydrogen otherwise known as Bohr's radius for hydrogen applies and n_x must $b_e = 1$.

Substitution of Eq. (3) into Eq. (4) gives

$$a_{\mathrm{X}} = \sqrt[2]{\frac{4\,\xi_{\mathrm{H}}}{\xi_{\mathrm{X}}}\frac{\pi\,\Gamma_{\mathrm{P}}}{m_{\mathrm{P}}}} \times \frac{n_{\mathrm{X}}\,h^3\,\varepsilon_0^{3/2}}{\mu_0\,e^5\,\pi} \tag{5}$$

It is also known [2] that for any hydrogenic and non-hydrogenic elements

$$a_{\mathrm{X}} = \frac{n_{\mathrm{X}} h}{\pi(8\,m_e)^{1/2}}\frac{1}{\xi_{\mathrm{X}}^{1/2}} \tag{6}$$

It follows from (5) and (6) that

$$\frac{1}{m_e} = 32\,\xi_{\mathrm{H}}\frac{\pi\,\mathcal{R}_{\mathrm{P}}}{m_{\mathrm{P}}}\frac{h^4\,\varepsilon_0^3}{\mu_0^2\,e^{10}} \tag{7}$$

From the Eq. (7) the final formula for Rp:

$$\mathcal{R}_{\mathrm{P}} = \frac{m_{\mathrm{P}}\,\mu_0^2\,e^{10}}{32\,h^4\,\varepsilon_0^3\,\pi\,m_e\,\xi_{\mathrm{H}}} \tag{8}$$

For every hydrogenic atom of the form ${}^{A}_{Z}X^{(Z-1)+}$ where, A and Z are the mass number and atomic number respectively, $n^2\xi_{(Z-1)}/Z^2$ (where n =1) as well as ($n_{\mathrm{X}}^2\xi_{\mathrm{X}}/Z_{\mathrm{eff}}^2$ is always = ξ_H where Z_{eff} is the effective nuclear charge. The constant in Eq. (8) is difficult to choose because although H is experimentally and theoretically determinable, it is nevertheless a constant parameter based on the preceding analysis. Thus, if all the parameters in Eq. (8) are constant, then the dependent parameter must be constant as long as the rest mass of the particle is constant.

Results and Discussion

As stated earlier, recent results reported for the length of the radii of nucleons in the literature [8] are very different from results reported in older literature [9, 10]. Such pieces of information include the observation that "the proton charge radii in the literature are 0.84 ± 0.05 fm [9], 0.856 fm [10], 0.84 ± 0.0004 fm [12-15], 0.84087 fm [16] and finally for this research, 0.831 ± 0.012 fm [17]. This latter figure for the proton charge radius refutes the claim that 0.84 fm may be the most accurate if lower figures point to more accurate values [8]. Yet, again the value of 0.831 ± 0.012 fm [17] may not be the shortest according to the paper [18] which showed a value equal to 0.7 fm.

Other reports are those based on an unfamiliar chiral bag model, which views the radius of the nucleon to be a bag of radius ≈ 0.8 fm [19] and that of [4], which is also 0.8 fm according to them, the r.m.s radius of the nucleon. According to the latter authors [4], proton point densities instead of charge densities can be used to determine the finite size of the proton by the prescription, $r_P^2 = r_C^2 - r_N^2$, where r_P, r_C, and r_N (where r_N is = 0.8 fm) are the r.m.s radii of proton, charge, and nucleon density distribution respectively.

The RP values for illustration are determined for the nucleons and muon in muonic hydrogen only as shown in Table 1. The muon is a lepton whose mass is ≈ 207 me, and its negative charge. The muon-proton mass ratio, η according to [20] is 0.1126095272.

Baryons	**Symbols**	**Mass/exp (-27) kg**	**Radii / fm**
Proton	p^+	1.672621777$^{(C)}$	1.101171175
Neutron	n	1.674927351$^{(C)}$	1.102689051
Leptons	**Symbols**	**Mass/exp (-27) kg**	**Radii / fm**
Muon	μ^-	0.188427357	0.123979254
Tauon	τ^-	3.167790098	2.085515799

In the Table 1 superscript (C) means the CODATA recommended values [21]. Other mass values use the mass-energy equivalence equation (GeV/c_2) with data in the www.Sciencedirect.com/topics/chemistry/leptons. The ionization energy of the hydrogen atom was calculated using a_0 = 5.2917721092·10^{-11} m, e =1.602176565·10^{-19} C, and ε_0 = 8.854187817·10^{-12} F/m. The usual equation is $\xi_H = e^2/8\pi\varepsilon_0 a_0$.

Based on Eq. (8), the radius of any elementary particle is mainly a function of its mass. Expectedly, the larger particle with higher mass has a longer radius similar to the report elsewhere [8]. Substitution of the ionization energy of hydrogen into Eq. (8) how-ever, gave results whose difference from previous results for the proton and neutron are ≈ 0.03406 % of the results in the literature [8].

In recent literature, elastic electron-proton scattering (e-p) and the spectroscopy of hydrogen atoms are the two methods traditionally used to determine the proton charge radius, Rp. Another method, using a muonic hydrogen atom, in which measurement of Lamp-shift was taken, found a substantial discrepancy compared with previous results [17]. The shorter length of the proton radius led to the proton radius puzzle [20]. Howev-er, a greater puzzle ought to be expected if despite the freely available literature materi-als the scholars did not notice the values of proton radius shorter than 0.831 fm. The proton radius in question was obtained by measuring the Lamb shift in the muonic hy-drogen atom.

The term Lamb shift came to reality after the notion of the degeneracy of $2s_{1/2}$

and $2p_{½}$ states of the hydrogen atom popularized by Dirac's unfamiliar one-particle relativistic theory was replaced with the observation that the $2s_{½}$ is higher than $2p_{½}$. Nevertheless, the potential energy at the p state should be higher than s. No extra information in this regard is available in the literature for clarification. Lamb shift is the shift of atomic energy level given suitable conditions, e.g. the interaction of the electron with the virtual photon and vacuum electric current (www.sciencedirect.com/topics/chemistry/lambshift), conditions that may not be unexpected in the experimental process involving muonic hydro-gen.

It has been explained [8] why the proton's r.m.s. radius will continue to shorten based on the implication of the experimental procedure, the unusual electron-proton scattering (e-p) approach involving particles of opposite charge as against the positronproton (e^+-p) scattering approach. The current approach in this research has dual characteristics in the sense that, as Eq. (8) shows, $\mathscr{R}_P$ being the massradius of any particle whose mass is equal to the mass of any of the nucleon or larger particles, is inversely proportional to the ionization energy of hydrogen atom, a parameter, which can be determined experimentally and theoretically. Besides, the Eq. (8) differs from Eq. (2) due largely to the presence of magnetic constant, μ02, as a nominator while Eq. (2) contains the velocity of light.

Conclusion

Nuclear properties such as the radius of any nucleon (RN) can be mathematically linked to atomic properties such as the ionization energy of hydrogen via an equation, which shows that RN is inversely proportional to the ionization energy of hydrogen and directly proportional to the rest mass of the particle. The chemistry of any atom is a function of its nuclear property. Thus, a link between hydrogenic and nuclear properties in furtherance of Bohr's theory is not out of place and it is not intended to be restricted to hydrogenic atoms as long as ionization energy is known.

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ՋՐԱԾՆԱՅԻՆ ԵՎ ՄԻՋՈՒԿԱՅԻՆ ՀԱՏԿՈՒԹՅՈՒՆՆԵՐԻ ԿԱՊԻ ՇՈՒՐՋ

Սերժ Պաշին

Արեգակի և երկրի ֆիզիկայի ինստիտուտ, Իրկուտսկ

Ամփոփագիր. Հետազոտության նպատակն է կապ հաստատել նուկլոնի զանգվածային շառավիղի ու ջրածնի իոնացման էներգիայի միջև, և որոշել նուկլոնների և լեպտոնների զանգվածային շառավիղները: Յանկացած տարրական մասնիկի շառավիղը կապված է ատոմական հատկությունների հետ, ինչպիսին է ջրածնի իոնացման էներգիան: Պարզվում է, այն հակադարձ համեմատական է ջրածնի իոնացման էներգիային և ուղիղ համեմատական մասնիկի հանգստի զանգվածին:

Հիմնաբառեր՝ ջրածնային իոն, նուկլոն, Բորի շառավիղ, ջրածնի իոնացման էներգիա:

К СВЯЗИ МЕЖДУ ВОДОРОДНЫМИ И ЯДЕРНЫМИ СВОЙСТВАМИ

Серж Пашин

ИСЗФ СО РАН

Целью исследования является установление связи между массовым радиусом нуклона и энергией ионизации водорода и определение массовых радиусов нуклонов и некоторых лептонов. Ядерный радиус любого нуклона связан с атомными свойствами, такими как энергия ионизации водорода, с помощью уравнения. Оказывается, радиусы элементарных частиц обратно пропорциональны энергии ионизации водоро-да и прямо пропорциональны массе покоя частиц.

Ключевые слова: водород, нуклон, радиус Бора, энергия ионизации водорода.